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FLEXIBLE REGRESSION EQUATION ALGORITHMS IN FORECASTING CHANGES IN THE HIX-LINDHAL INDEX

АЛГОРИТМИ ГНУЧКИХ РІВНЯНЬ РЕГРЕСІЇ У ПРОГНОЗУВАННІ ЗМІН ІНДЕКСУ ХІКСА-ЛІНДАЛЯ

In the modern world, rapid changes in economic, financial, and social conditions require the development of accurate and effective forecasting methods. One of the important tools for evaluating economic processes is forecasting changes in specific indices, such as the Hicks-Lindahl index (HIX-Lindahl index), which reflects a comprehensive indicator of harmonious development. Thus, the outlined article explores the effectiveness of applying flexible regression equation algorithms in forecasting changes in the Hicks-Lindahl index. The study concludes that the flexible regression equation algorithms used for forecasting changes in the HIX-Lindahl index are formed using methods and techniques of statistical analysis aimed at creating accurate forecasting models. Flexibility is provided by an equation that introduces regularization conditions to allow the model to adapt to complex, nonlinear relationships between variables.

Ключові слова: *independent variables, HIX-Lindahl index, triple effect, harmonious development, algorithm, flexible regression equations.*

У сучасному світі стрімкі зміни економічних, фінансових та соціальних умов зумовлюють потребу в розробці точних і ефективних методів прогнозування. Важливим інструментом для аналізу економічних процесів є прогнозування змін окремих індексів, зокрема індексу Хікса-Ліндаля, який виступає комплексним показником гармонійності розвитку. Таким чином, ця стаття присвячена дослідженню ефективності застосування алгоритмів гнучких рівнянь регресії для прогнозування змін індексу Хікса-Ліндаля, що є важливим інструментом оцінки добробуту в умовах змінних економічних реалій. У процесі дослідження застосовуються методи аналізу, синтезу, економіко-математичного моделювання та регуляції, які дають змогу детально оцінити механізми впливу різних факторів на індекс. Крім того, передбачається розробка практичних рекомендацій з оптимізації прогнозування змін цього індексу може бути корисною для комплексних наукових досліджень у сфері економіки. Результати дослідження підкреслюють, що алгоритми гнучких рівнянь регресії, застосовувані для прогнозування змін індексу Хікса-Ліндаля, розробляються із використанням методів і технік статистичного аналізу, спрямованих на створення точних моделей прогнозування. Гнучкість моделі забезпечується не лише самим рівнянням, але й запровадженням умов для його регуляризації, що дозволяє адаптувати модель до складних і нелінійних зв'язків між змінними. Дослідження доводять, що регресійні моделі з регуляризацією здатні зменшувати ризик перенавчання та покращувати узагальнення результатів, особливо за умови наявності великої кількості ознак або складних нелінійних взаємозв'язків між змінними. Наголошено, що найпоширенішими типами регуляризації регресійних моделей, які пояснюють поведінку індексу Хікса-Ліндаля, є L1-регуляризація (Lasso) та L2-регуляризація (Ridge). Констатовано, що регуляризація додає додатковий «штраф» до функції доходів, що змушує модель уникати надмірної залежності від окремих ознак, обмежуючи їхні ваги або коригуючи параметри для зменшення впливу варіативності в навчальній вибірці. Перспективами подальших досліджень є вдосконалення методів регуляризації в регресійних моделях для більш точного прогнозування змін індексу Хікса-Ліндаля в умовах високої складності і нелінійних зв'язків між змінними.

Keywords: *незалежні змінні, індексу Хікса-Ліндаля, триєдиний ефект, гармонійний розвиток, алгоритм, гнучкі рівняння регресії.*

Problem statement. In the modern world, rapid changes in economic, financial, and social conditions require the development of accurate and effective forecasting methods [3–4]. One of the important tools for assessing economic processes is the forecasting of changes in specific indices, such as the Hicks-Lindahl index (HIX-Lindahl index), which reflects a comprehensive indicator of development harmony. Accurate forecasting of such indices is critically important for making effective decisions at the level of individual enterprises [4; 5]. One of the approaches to forecasting is the use of regression methods, particularly flexible regression equation algorithms, which allow for the detection of complex relationships between variables and enable precise predictions even in conditions of high uncertainty.

Flexible regression equations, such as elastic nets or regularized models, provide the ability to avoid the problem of constant model transformation when working with large datasets and many independent variables.

These models are particularly relevant in forecasting financial indices, where numerous influences may change over time.

Analysis of research and publications. The issue of harmonious development of an enterprise and the formation of approaches to considering its components has been the focus of numerous works by both domestic and foreign scholars. Among them, it is worth noting researchers such as Todoryuk S. I., Kutarenko N. Ya. [6], Krysvatyy A. I. [5], Yudina O. I., and Klymova T. V. [7]. The merit of the scholars lies in their study of various aspects of economic, social, and managerial processes within an enterprise, the impact of internal and external factors on the components of the triple effect, as well as the processes of capital preservation and the improvement of the HIX-Lindahl net flow index.

However, an important element in considering these aspects is the effectiveness of applying flexible regression equation algorithms, particularly in forecasting changes in the HIX-Lindahl index. These algorithms allow for the assessment of how changes in one of the economic indicators can affect the harmonious development of the enterprise as a whole.

The results interpretation from such models opens up new opportunities for optimizing management decisions and improving the effectiveness of forecasting economic changes.

This article aims to explore the effectiveness of applying flexible regression equation algorithms in the context of forecasting changes in the HIX-Lindahl Index.

Presentation of the main material. Within the framework of the study, the author emphasizes that the HIX-Lindahl index can be considered as an effective indicator reflecting the pace and level of harmonious development of an enterprise, and is formed under the influence of complex factors. Given the dependent nature of the outlined indicator, flexible regression equation algorithms are of great importance for forecasting its variability.

It is important to note that flexible regression equation algorithms in forecasting changes in the HIX-Lindahl index are a set of methods and techniques for statistical analysis aimed at constructing accurate forecasting models through flexible (variable) regression equations.

In other words, these algorithms are important because they allow for the effective evaluation and forecasting of the behavior of any index that measures the complex change in a set of indicators (in our case, the triple effect of sustainable development), taking into account the various factors that influence them.

It should be noted that among the core elements of such algorithms are regression models that are adaptive to complex, nonlinear relationships between variables, as well as the HIX-Lindahl Index itself.

The HIX-Lindahl Index is calculated based on the aggregated indicator of the triple effect of sustainable development.

In this case, the basis for forecasting changes is a single mathematical expression of the regression [4; 7–8]:

$$HIXt=\beta 0+\beta 1\cdot X\ economic+\beta 2\cdot X\ social+ +\beta 3\cdot X\ environmental+\epsilon, \quad (1);$$

where: HIXt — the HIX-Lindahl Index at time t; X economic, X social, X environmental — the corresponding factors that characterize the economic, social, and environmental aspects; β0 — the constant (intercept); β1,β2,β3 — coefficients indicating the impact of each factor on the index; ϵ — the model's error term.

For example, for the company EKTA, one of the largest Eastern European developers and manufacturers of LED displays and business solutions based on LED technology, the model of variability of the HIX-Lindahl Index is formed by:

– the value of the economic capital preservation index X1 (in particular, monetary);

– the social capital preservation index X2 (in particular, human capital);

– the natural capital preservation index X3.

The basis for forecasting changes is the equation, which takes the form:

$$Y = 0.1965 + 2.0009X1 + +0.353X2-1.4953X3, \quad (2);$$

The input data for forming the equation is presented in Table 1.

For each of these models, it is important to formulate a hypothesis regarding the sensitivity of the given structure, which determines the variability of the HIX-Lindahl Index to changes.

For example, for the EKTA enterprise, it has been established that in the studied situation, 92.82% of the total variability of Y is explained by the change in the factors Xj.

Table 1

Input Data for Forming the Equation of Variability of the HIX-Lindahl Index for EKTA Enterprise, 2020–2024

Hicks-Lindahl index	Monetary Capital Preservation Index	Human Capital Preservation Index	Natural Capital Preservation Index
0,9	0,67	0,381	0,55
0,96	0,69	0,29	0,47
0,69	0,55	0,31	0,48
0,73	0,52	0,39	0,43
0,76	0, 67	0,43	0,6

Source: compiled based on EKTA Vision GmbH data

The economic interpretation of the model parameters is as follows:

– an increase in X1 by 1% leads to an average increase in Y by 2.001%;

– an increase in X2 by 1% leads to an average increase in Y by 0.353%;

– an increase in X3 by 1% leads to an average decrease in Y by 1.495%.

Ideally, this hypothesis is accepted or rejected based on the analysis of how different variables respond to internal and external factors, influencing harmonious development as a whole. The hypothesis tests the conditions of the index's sensitivity to changes in the external environment and internal processes, whose understanding allows for forecasting future development trends at different levels.

It should be noted that in order to accept or reject the proposed hypothesis, not just ordinary regression models should be considered, but rather models with regularization (as they are adaptive to complex, nonlinear relationships between variables) [5–7].

The point is that regression models with regularization help reduce overfitting and improve the generalization of the data when there are a large number of features or when complex nonlinear relationships exist between variables. Regularization adds an additional "penalty" (or regularization term [2; 3]) to the objective function, which forces the model to not overly rely on individual features, limiting their weights or adjusting the parameters so that the model is less prone to variability in the training dataset.

The most common types of regularization for regression models that explain the behavior of the HIX-Lindahl Index are L1 regularization (Lasso) and L2 regularization (Ridge), the characteristics of which are provided in Table 2. Another well-known method is Elastic Net regularization, which combines L1 and L2 regularization, allowing the model to adapt to complex relationships between variables that may be either linear or nonlinear.

The popularity of Lasso and Ridge models is due to their ability to reduce the impact of "noise" and avoid overfitting (when a model performs well on training data but poorly on new, unseen data) while simultaneously identifying the important features of the data.

Additionally, in modern forecasting of changes in the HIX-Lindahl Index, regression models using machine learning can be applied, which are effective due to their ability to adapt to nonlinear relationships between the index as the outcome variable and the numerical predictors.

Additionally, it is worth noting their value in identifying non-obvious patterns in large datasets that model the variability of the HIX-Lindahl index. Among these models, decision trees and neural networks are particularly important, as they can handle large volumes of data and provide high prediction accuracy, even in cases where traditional linear models fail to achieve good results (Table 3).

Machine learning methods, particularly decision trees and neural networks, significantly enhance the effectiveness of forecasting and data analysis due to their ability to process large volumes of data, detect nonlinear relationships, and reduce the impact of noise [3; 6].

At the same time, in practice, any of the regression models that are adaptive to complex, nonlinear relationships between variables provide the capability to handle sufficient data sets, uncover non-obvious dependencies, and reduce the influence of noise.

So, using the example of the variability equation for the HIX-Lindahl Index of the EKTA company (Y = 0.1965 + 2.0009X1 + 0.353X2 - 1.4953X3), its regularization is possible:

1. Using the Lasso model, the equation would look like:

Y = 0.1965 + 2.0009X1 - 1.4953X3 (since β2 has a weak influence on the forecast due to significant automation in the production of LED displays, and regularization has set it to zero (β2 = 0)).

Economic interpretation: an increase in X1 by 1% still leads to a 2.0009% increase in Y; the

Table 2

Characteristics of Lasso and Ridge models, adaptive to complex, nonlinear relationships between variables

Type of Regularized Model	Specificity of effectiveness in forecasting changes	Algorithm for model formation	Significance in Forecasting Changes in the Hicks-Lindahl Index
Lasso Regression (L1 Regularization)	Adds a regularization term to the cost function, proportional to the sum of the absolute values* of the model's coefficients.	$L1 = \Delta xi + \lambda i \sum_{n}^{1} \lvert wi \rvert$ where: Δxi — measure of error; wi — model coefficients (feature weights); λ — regularization parameter.	The regularization parameter helps minimize the prediction error and limit the magnitude of the model's coefficients. This means that the model will try to make the coefficients closer to zero. If the coefficients for certain features become zero, those features can be considered unnecessary for the model and can effectively be excluded from the analysis. This helps reduce the model's complexity and avoid overfitting.
Ridge Regression (L2 Regularization)	Adds a regularization term to the cost function, proportional to the sum of the squares of the coefficients	$L2 = \sum_{n=0}^{\infty} yi - (yi)^2 + \lambda \sum_{n=0}^{\infty} wj$ where: wj — model coefficients (feature weights); λ — regularization parameter, which controls the magnitude of the adjustment (the larger the λ, the stronger the penalty added to the cost function).	The regularization parameter is proportional to the sum of the squares of the model's coefficients. This means that an additional term is added to the cost function, which forces the model's coefficients to be as small as possible, but without eliminating them (unlike in Lasso regression, where some coefficients can be reduced to zero).

Note

** This variable is added to the cost function to limit the magnitude of the model's coefficients, preventing overfitting.*

Source: compiled based on [1–3; 6]

Table 3

Characteristics of decision Tree and Neural Network models, adaptive to complex, nonlinear relationships between variables

Type of machine learning model	Specificity of effectiveness in forecasting	Model Formation Algorithm	Significance in forecasting changes in the Hicks-Lindahl Index
Decision Trees	It splits the dataset into smaller subsets using conditional checks based on feature values. This allows the construction of a hierarchical model that makes decisions about how the relationship between variables should be structured.	At each stage, the operator selects the best variable (feature) that minimizes uncertainty the most (this is done through criteria such as information gain or mean squared error).	It is possible to clearly see how and why the model made a particular decision.
Neural Networks	They simulate the functioning of the human brain and are capable of detecting complex patterns in data. They consist of multiple layers of neurons that process the input data and pass it through several levels to produce the output.	Neural networks consist of input, hidden, and output layers. Each neuron in a layer receives input data, applies weights to it, passes it through an activation function, and then transmits the result to the next layer.	Through multiple hidden layers, neural networks can detect highly complex and nonlinear relationships between variables.

Source: compiled based on [1–2; 5; 8]

exclusion of X2 from the model means that the index of social capital preservation (X2) does not have a significant impact on Y. Meanwhile, the influence of X3 remains unchanged—its increase by 1% reduces Y by an average of 1.4953%.

Clearly, after Lasso regularization, the equation becomes simpler and focuses only on the most significant variables.

2. Using the Ridge model, the coefficients are constrained (β1 = 1.8, β2 = 0.3, β3 = -1.2), which reduces their influence but does not set

them to zero. Therefore, with the Ridge model, the equation would look like:

$$Y = 0.1965 + 1.8X1 + 0.3X2 - 1.2X3, \quad (3);$$

Economic interpretation: An increase in X1 by 1% leads to an increase in Y by 1.8% (a reduced impact compared to the initial model); an increase in X2 by 1% results in a 0.3% increase in Y (slightly lower than in the original model, but still significant); an increase in X3 by 1% reduces Y on average by 1.2% (a reduced negative impact).

The Ridge model is less prone to overfitting since the coefficients have become less extreme.

Conclusions. The study emphasizes that the algorithms of flexible regression equations used to forecast changes in the Hix-Lindahl index are formed using complex methods and statistical analysis techniques aimed at creating accurate forecasting models. The flexibility is ensured by the equation itself and by introducing conditions for its regularization, which allows the model to adapt to complex, nonlinear relationships between variables.

It has been proven that regression models with regularization can reduce overfitting and improve the generalization of results when there is a large number of features or complex nonlinear relationships between variables. Attention is drawn to the fact that the most common types of regularization in regression models, which explain the behavior of the Hix-Lindahl index, are L1 regularization (Lasso) and L2 regularization (Ridge).

It is stated that regularization adds an additional "fine" to the objective function, which forces the model to avoid excessive reliance on individual features by limiting their weights or adjusting parameters to reduce the impact of variability in the training sample.

The prospects for further research lie in improving regularization methods in regression models for more accurate forecasting of changes in the HIX-Lindahl Index under conditions of high complexity and nonlinear relationships between variables.

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